LISTA 13

Exercício 1.
Determine duas soluções l.i. pelo método de séries de potências:

(a) $y'' - xy' + x^2y = 0.$

(b) $y'' + y' - xy = 0.$

(c) $(1 + 2x^2)y'' + y' + (1 + x^2)y = 0.$

(d) $y'' - x\,y' - y = 0.$

(e) $y'' - x\,y' - 2\,y = 0.$

(f) $y'' + xy = 0$ (Equação de Airy)

(g) $(1 + 4x^2)y'' - 8y = 0$

(h) $(x^2 + 4)y'' + 2xy' - 12y = 0$

Exercício 2.
Resolva os problemas de valor incial abaixo pelo método das séries de potências, encontrando até o termo de ordem 5:

(a) $y'' - (1 + x^2)\,y = 0$, $y(0) = -2$, $y'(0) = 2.$

(b) $xy'' - y' + xy = 0$, $y(2) = 1$, $y'(2) = 2.$

(c) $y'' + xy' + x^2y = 0$, $y(-1) = 3$, $y'(-1) = -1.$

(d) $xy'' + 2\,y' - x^2y = 0$, $y(2) = 1$, $y'(2) = -1.$

(e) $y'' - (2x + 5)y' - y = 0$, $y(-2) = 1$, $y'(-2) = 1.$

(f) $xy'' - (x - 2)y' - y = 0$, $y(2) = -1$, $y'(2) = 1.$

(g) $y'' + (\operatorname{sen} x)\,y' + (\cos x)\,y = 0$, $y(0) = 2$, $y'(0) = 1.$

(h) $x^2y'' + (1 + x)\,y' + 3(\ln x)\,y = 0$, $y(1) = 2$, $y'(1) = 0.$

RESPOSTAS

1. (a) $y_1(x) = 1 - \frac{x^4}{12} - \frac{x^6}{90} + \frac{x^8}{3360} + \cdots$, $y_2(x) = x + \frac{x^3}{6} - \frac{x^5}{40} - \frac{x^7}{144} + \cdots$, $x \in \mathbb{R}\,.$

1. (b) $y_1(x) = 1 + \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{5\,x^6}{6!} + \cdots$, $y_2(x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{4\,x^5}{5!} + \frac{8\,x^6}{6!} + \cdots$, $x \in \mathbb{R}$.

1. (c) $y_1 = 1 - \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{12} + \cdots$, $y_2 = x - \frac{x^2}{2} + \frac{5\,x^4}{24} + \cdots$

1. (d) $y_1 = 1 + \frac{x^2}{2} + \frac{x^4}{2 \cdot 4} + \frac{x^6}{2 \cdot 4 \cdot 6} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!\,2^n}$, $x \in \mathbb{R}$,

$$y_2 = x + \frac{x^3}{3} + \frac{x^5}{3 \cdot 5} + \frac{x^7}{3 \cdot 5 \cdot 7} + \cdots = \sum_{n=0}^{\infty} \frac{1}{(2\,n+1)!!}\, x^{2\,n+1} = \sum_{n=0}^{\infty} \frac{n!\,2^n}{(2\,n+1)!}\, x^{2\,n+1}.$$

1. (e) $y_1 = 1 + \sum_{n=1}^{\infty} \frac{x^{2n}}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$, $y_2 = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!\,2^n} = x\,\exp\left(\frac{x^2}{2}\right)$, $x \in \mathbb{R}$.

1. (f) $y_1 = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n\,x^{3n}}{2 \cdot 5 \cdot 8 \cdots (3n-1) \cdot 3^n \cdot n!}$, $y_2 = x + \sum_{n=1}^{\infty} \frac{(-1)^n\,x^{3n+1}}{4 \cdot 7 \cdot 10 \cdots (3n+1) \cdot 3^n \cdot n!}$, $x \in \mathbb{R}$.

1. (g) $y_1 = 1 + 4x^2$, $x \in \mathbb{R}$ $y_2 = x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n} x^{2n+1}}{4n^2 - 1}$, $|x| < \frac{1}{2}$.

1. (h) $y_1 = 1 + 3 \sum_{n=1}^{\infty} \frac{(-1)^n (n+1) x^{2n}}{2^{2n}(2n-1)(2n-3)}$, $|x| < 2$, $y_2 = x + \frac{5}{12} x^3$, $x \in \mathbb{R}$.

2. (a) $y = -2 + 2x - x^2 + \frac{x^3}{3} - \frac{x^4}{4} + \frac{7x^5}{60} + \cdots$

2. (b) $y = 1 + 2(x-2) - \frac{5}{12}(x-2)^3 - \frac{1}{32}(x-2)^4 + \frac{13}{480}(x-2)^5 + \cdots$

2. (c) $y = 3 - (x+1) - 2\,(x+1)^2 + \frac{2}{3}(x+1)^3 + \frac{1}{4}(x+1)^4 - \frac{7}{30}(x+1)^5 + \cdots$

2. (d) $y = 1 - (x-2) + \frac{3}{2}(x-2)^2 - \frac{3}{4}(x-2)^3 + \frac{1}{2}(x-2)^4 - \frac{1}{5}(x-2)^5 + \cdots$

2. (e) $y = 1 + (x+2) + (x+2)^2 + \frac{5}{6}(x+2)^3 + \frac{5}{8}(x+2)^4 + \frac{5}{12}(x+2)^5 + \cdots$

2. (f) $y = -1 + (x-2) - \frac{1}{4}(x-2)^2 + \frac{5}{24}(x-2)^3 - \frac{1}{12}(x-2)^4 + \frac{11}{240}(x-2)^5 + \cdots$

2. (g) $y = 2 + x - \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{3}x^4 + \frac{1}{10}x^5 + \cdots$

2. (h) $y(x) = 2 - (x-1)^3 + \frac{5}{4}(x-1)^4 - \frac{27}{20}(x-1)^5 + \cdots$