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BOOK OF ABSTRACTS

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Porto Alegre
December 2020

Program

Monday, December 07

- 14h00 Jaqueline Siqueira (UFRJ)
- 15h00 Pablo Guarino (UFF)
- 16h00 Gabriela Estevez (UFRJ)

Tuesday, December 08

- 14h00 Jairo Bochi (PUC-Chile)
- 15h00 Silvius Klein (PUC-Rio)
- 16h00 Matías Carrasco (Udelar-Uruguai)

Abstracts

Global planar dynamics with star nodes: beyond Hilbert's 16th problem - PALESTRA CANCELADA

Begoña Alarcón (UFF)

Abstract: Global planar dynamics of polynomial vector fields has been of interest for many years. Part of this interest arises from its connection to Hilbert's 16^{th} problem on the number of limit cycles for the dynamics. Because of Hilbert's 16^{th} problem, substantial effort has been devoted to establishing a bound for the number of limit cycles. For some contributions in this direction when the vector field has homogeneous nonlinearities see the work of Huang *et al.* [7], Gasull *et al.* [6], Llibre *et al.* [8] or Carbonell and Llibre [3]. This question has also been approached using bifurcations by, for instance, Benterki and Llibre [2] or [6]. Problems with symmetry appear in Álvarez *et al.* [1].

We are, of course, also concerned in establishing an upper bound for the number of limit cycles. However, when no limit cycle exists, we take a different route and address the question of the existence of poly-cycles and the number of equilibria in them.

We focus on polynomial vector fields with homogeneous nonlinearities of type

$$\begin{cases} \dot{x} &= \lambda x + Q_1(x,y) \\ \dot{y} &= \lambda y + Q_2(x,y) \end{cases} \tag{0.1}$$

where $\lambda \neq 0$ and each Q_i, $i = 1, 2$ is a homogeneous polynomial of the same degree n. The origin of such a system is a star node.

We discuss the number and stability of equilibrium points, both in the plane and in the circle at infinity in the Poicaré compactification. We obtain conditions for the existence of a globally attracting poly-cycle, thus extending previous results on the existence of limit cycles. A more detailed analysis is done for symmetric vector fields.

The new approach of this paper is to describe the global planar dynamics making use of an invariant sphere for the dynamics, when it exists. The existence of such an invariant sphere is guaranteed by Field's [5, Theorem 5.1] Invariant Sphere Theorem and occurs quite naturally in some settings.

A classification of systems with $n = 3$ and $n = 2$ is also given, extending a previous work of A. Cima *et al.* [4] to our type of systems.

This is a joint work with Isabel Labouriau and Sofia Castro from Universidade do Porto (Portugal).

Referências

[1] M.J. Álvarez, A. Gasull and R. Prohens, Limit cycles for cubic systems with a symmetry of order 4 and without infinite critical points, *Proceedings of the AMS*, **136**(3), 1035–1043 (2008).

[2] R. Benterki and J. Llibre, Limit cycles of polynomial differential equations with quintic homogeneous nonlinearities, *Journal of Mathematical Analysis and Applications*, **407**, 16–22, (2013).
[3] M. Carbonell and J. Llibre. Limit Cycles of Polynomial Systems with Homogeneous Nonlinearities, *J. Math. Analysis and App.*, **142**, 573–590, (1989).
[4] A. Cima and J. Llibre. Algebraic and Topological Classification of the Homogeneous Cubic Vector Fields in the Plane. *J. Math. Analysis and App.*, **147**, 420–448, (1990).
[5] M.J. Field, Equivariant bifurcation theory and symmetry breaking, *J. Dynamics and Differential Equations*, **1**, 369–421, (1989).
[6] A. Gasull, J. Yu and X. Zhang, Vector fields with homogeneous nonlinearities and amny limit cycles, *Journal of Differential Equations*, **258**, 3286–3303, (2015).
[7] J. Huang, H. Liang and J. Llibre, Non-existence and uniqueness of limit cycles for planar polynomial differential systems with homogeneous nonlinearities, *Journal of Differential Equations*, **265**, 3888–3913, (2018).
[8] J. Llibre, J. Yu and X. Zhang, On the limit cycles of the Polynomial Differential Systems with a Linear Node and Homogeneous Nonlinearities, *International Journal of Bifurcation and Chaos*, **24**(5), 1450065–1–7, (2014).

Some recent results on multicritical circle maps

Gabriela Estevez (UFRJ)

Abstract: We study circle maps with a finite number of "inflexive"critical points, the so-called multicritical circle maps. The topology of these maps is well understood. One of the main questions in one dimensional dynamics is on the conditions that make the topology determine the geometry. In this talk, we will discuss some recent results concerning this question for these circle maps.

Flexibility of Lyapunov exponents

Jairo Bochi (PUC-Chile)

Abstract: I will sketch the flexibility program proposed by Anatole Katok, focusing on flexibility of Lyapunov exponents of conservative smooth diffeomorphisms. I will present results in this direction, obtained jointly with Katok, Federico Rodriguez Hertz, and Daren Wei.

On equilibrium states for impulsive semiflows

Jaqueline Siqueira (UFRJ)

Abstract: Impulsive dynamical systems may be interpreted as suitable mathematical models of real world phenomena that display abrupt changes in their behavior, and are described by three objects: a continuous semiflow on a metric space X; a set D contained in X; where the flow experiments sudden perturbations; and

an impulsive function $I : D \to X$; which determines the change on a trajectory each time it collides with the impulsive set D.

We consider impulsive semiflows defined on compact metric spaces and give sufficient conditions, both on the semiflows and the potentials, for the existence and uniqueness of equilibrium states. We also generalize the classical notion of topological pressure to our setting of discontinuous semiflows and prove a variational principle.

Based on some joint works with J. Alves, E. Bonotto, M. de Carvalho and S. Afonso.

Conformal dimension of hyperbolic groups that split over elementary subgroups

Matías Carrasco (Udelar-Uruguai)

Abstract: The conformal dimension of a hyperbolic group is a quasi-isometry invariant introduced by Pansu in the '80s. In this talk I will focus on the (Ahlfors regular) conformal dimension of the boundary at infinity of Gromov hyperbolic groups which split over elementary subgroups.

If such a group is not virtually free, I will show that the conformal dimension is equal to the maximal value of the conformal dimension of the vertex groups, or 1, whichever is greater. As a consequence, we can characterise which Gromov hyperbolic groups (without 2-torsion) have conformal dimension 1. This is a joint work with John M. Mackay.

Quasisymmetric orbit-flexibility

Pablo Guarino (UFF)

Abstract: In this talk we will discuss the following dynamical notion: two given orbits of a minimal circle homeomorphism f are said to be geometrically equivalent if there exists a quasisymmetric circle homeomorphism identifying both orbits and commuting with f. By a well-known theorem due to Herman and Yoccoz, if f is a smooth diffeomorphism with Diophantine rotation number, then any two orbits are geometrically equivalent. As it follows from the a-priori bounds of Herman and Swiatek, the same holds if f is a critical circle map with rotation number of bounded type. By contrast, in collaboration with Edson de Faria (Universidade de São Paulo), we recently proved that if f is a critical circle map whose rotation number belongs to a certain full Lebesgue measure set in $(0, 1)$, then the number of equivalence classes is uncountable. The proof of this result relies on the ergodicity of a two-dimensional skew product over the Gauss map. If there is enough time, we will show how, as a by-product of our techniques, we were able to construct topological conjugacies between multicritical circle maps which are not quasisymmetric, and how we show that this phenomenon is abundant, both from the topological and measure-theoretical viewpoints.

Uniform convergence rate for Birkhoff means of certain uniquely ergodic toral maps

Silvius Klein (PUC-Rio)

Abstract: The first goal of this talk is to present a Fourier analytical proof of (an extension of) the Denjoy-Koksma inequality, as well as an example showing its sharpness. While more technical than the original proof of Herman, our approach has the advantage of being easily adapted to other related contexts. Thus we obtain an estimate on the uniform convergence rate of the Birkhoff averages of a higher dimensional torus translation given by a frequency satisfying a generic arithmetic condition and a continuous observable. This convergence rate depends explicitly on the modulus of continuity of the observable and on the arithmetic properties of the frequency. Furthermore, we obtain similar results for affine skew product toral transformations. (Joint work with Xiao-Chuan Liu and Aline Melo.)