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Time Series Analysis

Correlated Errors in the Parameters Estimation of the ARFIMA Model: A Simulated Study

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Processes with correlated errors have been widely used in economic time series. The fractionally integrated autoregressive moving-average processes—ARFIMA(p, d, q)—(Hosking, 1981) have been explored to model stationary and non stationary time series with long-memory property. This work uses the Monte Carlo simulation method to evaluate the performance of some parametric and semiparametric estimators for long and short-memory parameters of the ARFIMA model with conditional heteroskedastic (ARFIMA-GARCH model). The comparison is based on the empirical bias and the mean squared error of each estimator.

Keywords Fractional differencing; Long memory and GARCH models.

Mathematics Subject Classification 62M10; 62M15.

1. Introduction

Recently, the analysis of time series with long memory has been widely argued in the literature. Since the ARFIMA(p, d, q) model was introduced by Granger and Joyeux (1980) and Hosking (1981), many estimation methods of the memory parameter d have been proposed and some corrections of the estimators have been suggested to improve their statistical properties. Amongst the usually so-called parametric approaches we mention the works of Fox and Taqqu (1986), Dahlhaus (1989), and Sowell (1992) which are estimation methods based on the maximum likelihood theory. Among the semiparametric approaches we cite the works of Geweke and Porter-Hudak (1983), Robinson (1994, 1995), Reisen (1994),

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and Lobato and Robinson (1996). Bootstrap estimation procedures for d has been the focus of Franco and Reisen (2004) and references therein. The estimation methods for memory parameters of seasonal fractionally ARMA models have been discussed and investigated empirically by Reisen et al. (2006a,b). In this article, the authors have also presented the invertibility and casual parameters conditions of the model.

The main goal of this article is to evaluate, through a simulation study, the bias and the robustness of six estimation methods for the fractional differencing parameter d in ARFIMA processes with heteroskedastic errors. This research topic has recently received considerable attention in a variety of studies in time series and econometric areas. Ling and Li (1997), Henry (2001a,b), and Jensen (2004) have provided a good survey of the literature. The ARFIMA-GARCH model and its parameter estimators have been considered in Ling and Li (1997). The authors have derived some sufficient conditions for stationarity and ergodicity, and an algorithm for approximate maximum likelihood estimation has been also presented. Henry (2001a) has focused his research on the estimation of the memory parameter of a time series with long-memory conditional heteroskedasticity (ARFIMA-GARCH model) by using the average periodogram estimator suggested in Robinson (1994). In this context, Henry (2001a) has shown that the average periodogram estimator remains consistent. This semiparametric memory parameter estimation method is also considered here. The choice of the optimal bandwidth for some semiparametric estimation methods, under a general form of the errors such as GARCH errors, has been the focus of Henry (2001b). The author has derived optimal bandwidths and has shown asymptotically that the estimators considered in his work are not affected by conditional heteroskedasticity property of the innovations.

Jensen (2004) has given contributions based on Bayesian approach to estimate the memory parameter in long-memory stochastic volatility processes. An application of the useful ARFIMA-GARCH model in inflation data has been the focus of Baillie et al. (1996).

The results reported here increase our understanding of the finite sample properties of some fractional memory parameter estimators of the ARFIMA model with heteroscedastic errors. The plan of this article is as follows. Section 2 outlines the use of ARFIMA model with GARCH errors. Section 3 presents a summary on the estimation methods for the parameter d. Section 4 discusses the set-up of the Monte Carlo experiment and the results. Some conclusions are drawn in Sec. 5.

2. The ARFIMA Model with GARCH Errors

Let $\{X_t\}_{t\in Z}$ be a time series presented in a set of information available at an instant $t-1$. If ψ_{t-1} is a set of evaluated available information at an instant $t-1$, we can represent $\{X_t\}_{t\in Z}$ by the form

$$X_t = g(\psi_{t-1}, \mathbf{b}) + \varepsilon_t, \tag{2.1}$$

where $g(\cdot\,,\cdot)$ is a function, $\mathbf{b}$ is a vector of the parameters to be estimated, and ε_t is a random perturbation. Equation (2.1) is sufficiently general and it has been studied and shaped by many authors. The most common specification for it is the *autoregressive* AR(p) *model* and the *moving average* MA(q) *model* that can be mixed to have the ARMA(p, q) model, described hereof by

$$\Phi(\mathscr{B})(X_t - \mu) = \Theta(\mathscr{B})\varepsilon_t, \quad t \in Z, \tag{2.2}$$

where $\mathscr{B}$ is the backward operator such that $\mathscr{B}^k X_t = X_{t-k}$, $\Phi(\mathscr{B}) = 1 - \sum_{j=1}^{p} \phi_j \mathscr{B}^j$, $\Theta(\mathscr{B}) = 1 - \sum_{i=1}^{q} \theta_i \mathscr{B}^i$, p and q are positive integers, μ is the mean of the process, and $\{\varepsilon_t\}_{t \in Z}$ is a white noise process with zero mean and constant variance σ_ϵ^2. $\Phi(\mathscr{B})$ and $\Theta(\mathscr{B})$ are polynomials with all roots outside the unit circle and share no common factors.

Studies in economic time series have shown that the dependent variable—returns in the interest rates, for instance—presents significant autocorrelation, even for lags largely separated in time. Time series with this behavior is said to have long-memory property and it may be modeled by the ARFIMA(p, d, q) model described as

$$\Phi(\mathscr{B})(1 - \mathscr{B})^d (X_t - \mu) = \Theta(\mathscr{B})\varepsilon_t, \quad t \in Z, \tag{2.3}$$

where d is a real number.

When $d \in (-0.5, 0.5)$, the ARFIMA(p, d, q) process is said to be invertible and stationary and its spectral density function $f_X(\cdot, \cdot)$ is given by

$$f_X(w, \zeta) = f_U(w, \xi)\left[2 \sin\left(\frac{w}{2}\right)\right]^{-2d}, \quad w \in [-\pi, \pi], \tag{2.4}$$

where $f_U(w, \xi)$ is the spectral density function of an ARMA(p, q) process and ξ and ζ are the unknown parameter vectors of the ARMA and ARFIMA models, respectively. Hosking (1981), Beran (1994), and Reisen (1994) have described the ARFIMA models with more details. Engle (1982) has defined the process *conditional autoregressive heteroskedastic* (ARCH) when ε_t is of the form

$$\varepsilon_t = z_t \sigma_t, \tag{2.5}$$

where z_t is an independent and identically distributed process with $E(z_t) = 0$ and $Var(z_t) = 1$ and σ_t, varying in time, is a function of the set ψ_{t-1}. By definition, the variables ε_t are not autocorrelated, for any $t \in Z$, but its conditional variance depends on time, opposing to what is assumed for the usual least squares estimation method. The ARCH(s) model or its generalization GARCH(s, r) model can be summarized here, through the form of the innovation variance for the time t, according to Eq. (2.1) and with the assumption that $\varepsilon_t | \psi_{t-1} \sim \mathscr{N}(0, \sigma_t^2)$, where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2, \tag{2.6}$$

where s and r are positive integers, $\alpha_i \geq 0$, for $i \in \{0, 1, \ldots, s\}$, and $\beta_j \geq 0$, for $j \in \{1, 2, \ldots, r\}$.

Bollerslev (1986) has shown that a GARCH process is stationary if $\alpha(1) + \beta(1) < 1$, where $\alpha(1) = \sum_{i=1}^{s} \alpha_i$ and $\beta(1) = \sum_{j=1}^{r} \beta_j$, whenever $E(\varepsilon_t) = 0$, $Var(\varepsilon_t) = \alpha_0/[1 - \alpha(1) - \beta(1)]$ and the innovation process is not autocorrelated. We remark here that if one excludes the last sum in Eq. (2.6), one has the simplest model, an ARCH(s) model.

The combination of models in expressions (2.3), (2.5), and (2.6) yields to the ARFIMA (p, d, q)-GARCH(r, s) model. Under the parameter conditions for $\Phi(\mathscr{B})$, $\Theta(\mathscr{B})$, $\alpha(1)$, and $\beta(1)$, as previously described, the ARFIMA(p, d, q)-GARCH(r, s) process is stationary and invertible if $-0.5 \leq d \leq 0.5$, see Theorem 2.3 in Ling and Li (1997). The authors have also demonstrated the condition for ergodicity and derived the existence of higher-order moments. To estimate the parameters, an algorithm for approximate maximum likelihood (ML) estimation has been also presented by the authors.

The autocorrelation function of ARFIMA(p, d, q)-GARCH(r, s) model behaves asymptotically with the similar form of the stationary ARFIMA model. This means that the dependency between observations decays hyperbolically as a function of d. Based on this autocorrelation behavior, we are here interested in studying the estimation of the parameter d for some parameter order specifications of the ARFIMA(p, d, q)-GARCH(r, s) model. This empirical study will provide an evidence whether the memory estimators are robust or not under conditional heteroskedasticity. It is worth noting that all estimation methods focused here have been previously implemented to the estimation of the stationary and non stationary ARFIMA processes see, for instance, Lopes et al. (2004) and Reisen et al. (2001a) for an overview.

3. Estimation of *d*

We consider six estimators for the parameter d and provide a summary of each method below. The first four approaches are based on the linear regression method built from Eq. (2.4) and are considered semiparametric methods. The averaged periodogram estimator also belongs to the semiparametric class. The remaining two estimators are the parametric methods proposed by Fox and Taqqu (1986) and Sowell (1992).

The first estimator, denoted hereafter by $\hat{d}_{Per}$, has been proposed by Geweke and Porter-Hudak (1983). They use the periodogram function $I(\cdot)$ as an estimator for the spectral density function, presented in Eq. (2.4). The number of regressors used in the regression equation is a function of the sample size n and it is denoted here by $g(n) = n^{\alpha}$, for $0 < \alpha < 1$.

The second estimator, denoted by $\hat{d}_{Per\,S}$, has been proposed by Reisen (1994). The author has made a modification in the regression equation, substituting the periodogram function by its smoothing version based on the Parzen lag window. In this estimator the function $g(n) = n^{\alpha}$, for $0 < \alpha < 1$, is also chosen to represent the number of regressors in the regression equation. The truncation point in the Parzen lag window is denoted by $m = n^{\beta}$, for $0 < \beta < 1$. The appropriate choices of α and β values had been investigated by Geweke and Porter-Hudak (1983) and Reisen (1994), respectively, among several other authors.

The third estimator, denoted hereafter by $\hat{d}_{Lob}$, has been considered by Robinson (1994) and Lobato and Robinson (1996). It is a weighted average of the logarithm of the periodogram function. This estimator is based on a number of frequencies τ and on a constant $q \in (0.0, 1.0)$. Lobato and Robinson (1996) have presented a Monte Carlo simulated experiment to investigate the sensitivity on the choice of τ and q values.

The fourth estimator, denoted by $\hat{d}_{Rob}$, has been introduced by Robinson (1995). This method is also a modified version of the Geweke and Porter-Hudak's estimator. This estimator regresses $\ln\{I(w_j)\}$ on $\ln\{2\sin(w_j/2)\}^2$, for $j = l, l+1, \ldots, g(n)$, where l is the smallest truncated point that tends to infinity smoother than $g(n)$. Robinson (1995) has developed some asymptotic results for $\hat{d}_{Rob}$, when $d \in (0.0, 0.5)$, and has showed that this estimator is asymptotically less efficient than the Gaussian maximum likelihood estimator and in this situation the efficiency is considered to be zero. The number of regressors $g(n)$ can take different forms and its appropriate choice has been studied by several authors, such as Robinson (1995), Hurvich et al. (1998), and Hurvich and Deo (1999), to name just a few.

The fifth estimator, hereafter denoted by $\hat{d}_{Fox}$, is a parametric procedure that has been considered by Fox and Taqqu (1986), based on the work of Whittle (1953). The estimates are obtained by minimizing the finite and discrete form given by

$$\mathcal{L}_n(\zeta) = \frac{1}{2n}\sum_{j=1}^{n-1}\left\{\ln f_X(w_j,\zeta) + \frac{I(w_j)}{f_X(w_j,\zeta)}\right\}, \tag{3.1}$$

where ζ denotes the vector of unknown parameters and $f_X(w,\zeta)$ is the spectral density function (see Dahlhaus, 1989; Fox and Taqqu, 1986).

The ML estimator (see Sowell, 1992) is the sixth method focused here. Assuming that the process $\{X_t\}$ is Gaussian, the log-likelihood function may be expressed as

$$\mathcal{L}_n(\zeta) = -\frac{1}{2}\log\det T(f_X(w,\zeta)) - \frac{1}{2}X'T(f_X(w,\zeta))^{-1}X, \tag{3.2}$$

where ζ is the parameter vector and T is the variance–covariance matrix of the process $\{X_t\}_{t\in Z}$ with (j,k)th element given by

$$T_{jk}(f_X(w,\zeta)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f_X(w,\zeta)\exp(iwjk)dw,$$

see for example Beran (1994, Sec. 5.3). The ML estimates are obtained by maximizing (3.2), that is, $\hat{\zeta} = \arg\max \mathcal{L}_n(\zeta)$. Following the estimator notations adopted here the ML fractional estimator is denoted by $\hat{d}_{ML}$.

Fractional integration has been applied in models like GARCH, including the FIGARCH model by Chung (1999), the FIEGARCH model by Baillie et al. (1994), and the FIAPARCH model by Tse (1998). All of them have in common some variations in the estimation process through the maximization of the likelihood function. The comparison among parameter estimation procedures have been widely studied and they are sources of research for several authors. However, it is not our main goal. This article focuses on the robustness of the estimators for the fractional parameter d, in view of violating the normality assumption for the innovation process and also in the study of how they are affected by the existence of heteroskedastic errors.

4. Monte Carlo Simulation Study

In order to investigate the robustness of the estimators described previously, in the context of ARFIMA models with GARCH errors, a number of Monte

Carlo experiments were carried out. Realization of the Gaussian white noise sequence z_t, defined in (2.5), $t = 1, \ldots, n$, with unit variance, were generated by IMSL-FORTRAN subroutine DRNNOR. The ARFIMA processes were generated according to Hosking (1984) with errors ε_t given by (2.5) and σ_t^2 by (2.6). The models and the parameter values are specified in the tables which also give the empirical mean and mean squared error (MSE) of the estimation methods based on 1,000 replications of series with length $n = 300$. For comparison purpose, the tables also give the estimation results when the errors of the ARFIMA process were generated from $\mathcal{N}(0, 1)$ distribution. In the semiparametric methods two bandwidths were used, and their values are also specified in the tables. The trimming number was fixed $l = 2$ for the $\hat{d}_{Rob}$ estimator, and the constant $q = 0.5$ for the averaged periodogram estimator. Note that we did not concentrate on the sensitivity of the estimate with the choice of the constant q. A review of this method and the appropriated choice of q are the main focus of Lobato and Robinson (1996). In the parametric group, the estimates of $\hat{d}_{Fox}$ were obtained by using the subroutine DBCONF-IMSL. For the same data set, the ML estimates were obtained by using the Ox-program (Doornik, 1999).

Here, the finite sample performance of the estimators described previously was examined under different structures of the ARFIMA-GARCH model with two memory parameter values: $d = 0.2$ (moderate long memory) and $d = 0.45$ (strong long memory). The choice of $d = 0.2$ is motivated by the theoretical results of the averaged periodogram estimator given in Lobato and Robinson (1996).

Results from ARFIMA(0, d, 0) model are in Tables 1a and b. From these tables we see that all methods perform well. When the process is generated by Gaussian

Table 1a
ARFIMA(0, d, 0) model with Gaussian GARCH errors, $d = 0.2$ and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
				$d = 0.2$, Gaussian error						
$\hat{d}$(mean)	0.2042	0.2017	0.1513	0.1806	0.1676	0.1652	0.2098	0.2030	0.1960	0.1796
MSE	0.0407	0.0106	0.0267	0.0074	0.0071	0.0046	0.0689	0.0131	0.0025	0.0028
				$d = 0.2$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.2$						
$\hat{d}$(mean)	0.1922	0.1966	0.1464	0.1814	0.1701	0.1669	0.1917	0.1973	0.1982	0.1842
MSE	0.0403	0.0101	0.0267	0.0071	0.0068	0.0047	0.0661	0.0133	0.0032	0.0032
				$d = 0.2$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.5$						
$\hat{d}$(mean)	0.2016	0.2004	0.1462	0.1804	0.1662	0.1641	0.1946	0.1981	0.1939	0.1836
MSE	0.0399	0.0106	0.0258	0.0079	0.0078	0.0068	0.0670	0.0137	0.0051	0.0051
				$d = 0.2$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.9$						
$\hat{d}$(mean)	0.1970	0.1823	0.1446	0.1648	0.1479	0.1403	0.1978	0.1800	0.1745	0.1783
MSE	0.0387	0.0177	0.0255	0.0138	0.0167	0.0170	0.0686	0.0235	0.0116	0.0132
				$d = 0.2$, GARCH error, with $\alpha_0 = 0.3$, $\alpha_1 = 0.2$, and $\beta_1 = 0.5$						
$\hat{d}$(mean)	0.2009	0.1974	0.1527	0.1807	0.1638	0.1668	0.1983	0.1960	0.1971	0.1810
MSE	0.0417	0.0118	0.0261	0.0080	0.0083	0.0053	0.0716	0.0149	0.0032	0.0034

Table 1b
ARFIMA(0, d, 0) model with Gaussian and GARCH errors, $d = 0.45$ and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
	$d = 0.45$, Gaussian error									
$\hat{d}$(mean)	0.4623	0.4592	0.4008	0.4394	0.3487	0.3552	0.4594	0.4581	0.4869	0.4194
MSE	0.0405	0.0101	0.0294	0.0070	0.0125	0.0105	0.0631	0.0128	0.0038	0.0024
	$d = 0.45$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.2$									
$\hat{d}$(mean)	0.4595	0.4608	0.4009	0.4384	0.3479	0.3540	0.4661	0.4627	0.4770	0.4178
MSE	0.0409	0.0092	0.0281	0.0066	0.0126	0.0107	0.0670	0.0114	0.0038	0.0028
	$d = 0.45$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.5$									
$\hat{d}$(mean)	0.4491	0.4605	0.3961	0.4394	0.3474	0.3542	0.4509	0.4628	0.4663	0.4155
MSE	0.0387	0.0115	0.0293	0.0086	0.0132	0.0112	0.0636	0.0150	0.0057	0.0041
	$d = 0.45$, ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.9$									
$\hat{d}$(mean)	0.4502	0.3945	0.4393	0.4196	0.3325	0.3354	0.4575	0.4396	0.4377	0.3959
MSE	0.0418	0.0300	0.0176	0.0136	0.0189	0.0188	0.0722	0.0229	0.0136	0.0100
	$d = 0.45$, GARCH error, with $\alpha_0 = 0.3$, $\alpha_1 = 0.2$, and $\beta_1 = 0.5$									
$\hat{d}$(mean)	0.4729	0.4592	0.4116	0.4388	0.3483	0.3559	0.4662	0.4551	0.4884	0.4178
MSE	0.0413	0.0118	0.0280	0.0086	0.0132	0.0106	0.0654	0.0148	0.0049	0.0029

errors, the biases are generally small, specially for $d = 0.2$. In the semiparametric class, the regression methods are very competitive where $\hat{d}_{Per\,S}$ is an estimator with smaller MSE. In general, the estimates from $\hat{d}_{Lob}$ present the largest biases, but with the smallest MSE among all semiparametric estimation methods. The MSE is small due to the substantial reduction of its sample variance. The increase of the bandwidth leads to estimates with small MSE. These findings are in accordance with other published works as in Lopes et al. (2004) and Reisen et al. (2001a,b) among others. The estimator $\hat{d}_{Fox}$ has typically smaller absolute bias than $\hat{d}_{ML}$ and their MSE are nearly identical. These results are consistent with the findings in Cheung and Diebold (1994). These authors have presented a comprehensive analysis comparing both parametric methods with unknown mean, see also Baillie et al. (1996). When $d = 0.45$ (Table 1b), the methods perform similarly to the case $d = 0.2$ (Table 1a) except for the average periodogram estimator. The absolute bias of this later method increased substantially and it may be explained by the fact that its asymptotic Normal distribution is only guarantied when $0 < d < 0.25$.

In general, it seems that there is no significant change in the estimates when considering the ARCH and GARCH error types. This evidences that the estimators are robust in the presence of heteroskedastic errors. For a process where the correlation of the squares are very strong (ARCH model with $\alpha_1 = 0.9$), there is an empirical evidence that the bias and the MSE of the parametric estimates increase substantially. By increasing the bandwidth, independently of the error type, the semiparametric estimates become more precise by presenting smaller MSE and are very competitive with the parametric methods. These empirical results reveal that the asymptotic normality results for the investigated estimation methods still hold for heteroskedastic errors. Similar findings using the average periodogram and parametric estimators are, respectively, in Henry (2001a) and Baillie et al. (1996).

Reisen et al. (2001a) have studied the robustness of these estimators when the errors come from a non normal distribution. Their empirical investigation have evidenced that the estimators have no influence with the violation of normality assumption, and have called the attention of the good performance of the parametric estimator $\hat{d}_{Fox}$ under correct model specification.

Tables 2–6 present the simulation results when short-memory parameters were introduced in the ARFIMA-ARCH model. Here we present the ARMA combinations $(\pm 0.7, 0)$, $(0, \pm 0.7)$, $(0.2, \pm 0.7)$, and $(\pm 0.7, \pm 0.2)$. Other parameter values produced similar behavior and are available upon request. The values of the true and the estimated short-memory parameters are also given in the tables. The $\hat{d}_{Fox}$ and $\hat{d}_{ML}$ methods estimates all parameters simultaneously. The estimates from the semiparametric methods were obtained from two steps: first the long-memory parameter estimate $\hat{d}$ was obtained then, the filter $(1 - B)^{\hat{d}}$ was applied to the series to obtain the short-memory parameter estimates. From the results we observe that the short-memory parameter affects in the parameter estimates. This is directly related to the type of ARMA model and to the sign and magnitude of the short-memory coefficients. The estimates are also affected by the size of the bandwidth. In general, the estimators seem to be unaffected by the presence of ARCH errors.

For the ARFIMA$(1, d, 0)$ model (see Tables 2 and 3) the study revels that the bias of d is affected by the sign and the value of the AR coefficient. Results for

Table 2
ARFIMA$(1, d, 0)$ model with Gaussian and ARCH errors, $d = 0.2$, $\phi_1 = -0.7, 0.7$, and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
Gaussian error, $d = 0.2$ and $\phi_1 = -0.7$										
$\hat{d}$(mean)	0.1960	0.1567	0.1422	0.1373	0.1224	−0.0182	0.1924	0.1486	0.1991	0.1754
MSE	0.0429	0.0120	0.0281	0.0107	0.0132	0.0540	0.0708	0.0158	0.0040	0.0036
$\hat{\phi}_1$	−0.6579	−0.6707	−0.6484	−0.6643	−0.6554	−0.5514	−0.6229	−0.6630	−0.7117	−0.6863
MSE	0.0278	0.0058	0.0157	0.0053	0.0067	0.0329	0.0667	0.0081	0.0093	0.0026
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, and $\phi_1 = -0.7$										
$\hat{d}$(mean)	0.1977	0.1571	0.1430	0.1380	0.1212	−0.0189	0.2016	0.1506	0.2043	0.1771
MSE	0.0405	0.0123	0.0276	0.0108	0.0133	0.0558	0.0641	0.0155	0.0055	0.0041
$\hat{\phi}_1$	−0.6585	−0.6693	−0.6471	−0.6633	−0.6538	−0.5473	−0.6342	−0.6632	−0.7371	−0.6858
MSE	0.0291	0.0065	0.0186	0.0059	0.0072	0.0357	0.0571	0.0082	0.0180	0.0028
Gaussian error, $d = 0.2$ and $\phi_1 = 0.7$										
$\hat{d}$(mean)	0.3226	0.5793	0.2735	0.5614	0.4016	0.4446	0.3512	0.6311	0.2754	0.1328
MSE	0.0583	0.1541	0.0292	0.1374	0.0416	0.0600	0.1011	0.1990	0.0379	0.0179
$\hat{\phi}_1$	0.5697	0.3326	0.6179	0.3487	0.5069	0.4659	0.5388	0.2840	0.6135	0.7363
MSE	0.0521	0.1455	0.0273	0.1307	0.0396	0.0569	0.0850	0.1856	0.0396	0.0113
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, and $\phi_1 = 0.7$										
$\hat{d}$(mean)	0.3380	0.5805	0.2773	0.5585	0.4009	0.4438	0.3801	0.6338	0.2517	0.1334
MSE	0.0584	0.1546	0.0304	0.1351	0.0412	0.0597	0.0960	0.2012	0.0329	0.0202
$\hat{\phi}_1$	0.5519	0.3276	0.6098	0.3475	0.5027	0.4619	0.5118	0.2780	0.6278	0.7321
MSE	0.0565	0.1499	0.0305	0.1325	0.0420	0.0593	0.0897	0.1914	0.0335	0.0118

Table 3
ARFIMA(1, d, 0) model with Gaussian and ARCH errors, $d = 0.45$, $\phi_1 = -0.7, 0.7$, and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
	Gaussian error, $d = 0.45$ and $\phi_1 = -0.7$									
$\hat{d}$(mean)	0.4660	0.4156	0.4079	0.3961	0.3249	0.2593	0.4580	0.4044	0.5412	0.4114
MSE	0.0428	0.0114	0.0292	0.0098	0.0186	0.0398	0.0626	0.0148	0.0126	0.0034
$\hat{\phi}_1$	−0.6531	−0.6688	−0.6354	−0.6616	−0.6177	−0.5513	−0.6225	−0.6579	−0.8905	−0.6823
MSE	0.0440	0.0068	0.0332	0.0064	0.0118	0.0336	0.0680	0.0102	0.0549	0.0024
	ARCH error, with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.45$, and $\phi_1 = -0.7$									
$\hat{d}$(mean)	0.4416	0.3993	0.3891	0.3840	0.3157	0.2437	0.4357	0.3897	0.4480	0.4137
MSE	0.0392	0.0129	0.0284	0.0140	0.0212	0.0456	0.0627	0.0170	0.0051	0.0035
$\hat{\phi}_1$	−0.6700	−0.6758	−0.6578	−0.6754	−0.6366	−0.5851	−0.6459	−0.6686	−0.7639	−0.6816
MSE	0.0624	0.0505	0.0525	0.0496	0.0496	0.0588	0.1061	0.0593	0.0708	0.0029
	Gaussian error, $d = 0.45$ and $\phi_1 = 0.7$									
$\hat{d}$(mean)	0.5806	0.8313	0.5233	0.8122	0.4622	0.4822	0.6098	0.8824	0.8659	0.3159
MSE	0.0611	0.1557	0.0331	0.1379	0.0004	0.0011	0.1013	0.2003	0.1965	0.0250
$\hat{\phi}_1$	0.5680	0.3287	0.6267	0.3465	0.6997	0.6802	0.5375	0.2807	0.2972	0.7891
MSE	0.0567	0.1489	0.0320	0.1327	0.0019	0.0023	0.0889	0.1890	0.2062	0.0120
	ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.45$, and $\phi_1 = 0.7$									
$\hat{d}$(mean)	0.5837	0.8285	0.5245	0.8109	0.4618	0.4820	0.6250	0.8821	0.7750	0.3205
MSE	0.0585	0.1538	0.0317	0.1369	0.0004	0.0011	0.0999	0.2001	0.1339	0.0245
$\hat{\phi}_1$	0.5687	0.3354	0.6262	0.3514	0.7007	0.6812	0.5264	0.2855	0.3955	0.7850
MSE	0.0556	0.1448	0.0314	0.1302	0.0023	0.0027	0.0887	0.1855	0.1557	0.0114

$\phi = \pm 0.2$ are available upon request. By increasing the bandwidth from $g(n) = n^{0.5}$, the semiparametric estimates overestimate the true parameter and the bias becomes significantly large. This overestimation gives estimate values in the non stationary region. There is an increasing of the MSE of AR estimates which is associated with the large sample variance of the AR estimate. The effect of positive AR coefficient in the estimates may be explained by the fact it produces large contribution to the spectral density for those frequencies away from the zero frequency. In the parametric group, positive and large AR parameter value seems to affect the estimates specially those from the ML estimator. In this method, the absolute bias increases substantially. For negative values of ϕ, the biases of both parameters (d and ϕ) are relatively small (negative bias for d and positive for ϕ) while, for positive ϕ there is a large positive bias for d and, consequently, large negative bias for ϕ. In general, for the process with strong memory ($d = 0.45$), there is a similar behavior when $d = 0.2$, however, its estimates strongly suggest a non stationary memory parameter value.

In general, the presence of heteroskedastic errors in the ARFIMA model does not affect the estimates. The biases seem to be intrinsically related to the AR parameter values.

Table 4 shows the results of ARFIMA(0, d, 1) model. For negative θ, it is observed similar behavior of the estimates from the previous model when $\phi < 0$. The biases are generally small and, the empirical values of MSE are nearly identical. In the semiparametric methods, the values of the bandwidth does not affect

Table 4
ARFIMA(0, d, 1) model with Gaussian and ARCH errors, $d = 0.2$, $\theta_1 = -0.7, 0.7$, and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
Gaussian error, $d = 0.2$ and $\theta_1 = -0.7$										
$\hat{d}$(mean)	0.2072	0.1567	0.2438	0.2250	0.2101	0.2857	0.2066	0.2504	0.2002	0.1806
MSE	0.0413	0.0281	0.0123	0.0076	0.0049	0.0091	0.0715	0.0158	0.0031	0.0033
$\hat{\theta}_1$	−0.6917	−0.6805	−0.7140	−0.6890	−0.6956	−0.6659	−0.6872	−0.6778	−0.6933	−0.7086
MSE	0.0096	0.0046	0.0062	0.0033	0.0026	0.0038	0.0173	0.0053	0.0026	0.0027
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, and $\theta_1 = -0.7$										
$\hat{d}$(mean)	0.2149	0.2478	0.1591	0.2266	0.2109	0.2846	0.2193	0.2554	0.1981	0.1821
MSE	0.0357	0.0114	0.0249	0.0070	0.0047	0.0091	0.0598	0.0150	0.0037	0.0037
$\hat{\theta}_1$	−0.6869	−0.6775	−0.7121	−0.6862	−0.6933	−0.6645	−0.6798	−0.6742	−0.6938	−0.7073
MSE	0.0102	0.0048	0.0060	0.0038	0.0032	0.0042	0.0179	0.0056	0.0027	0.0024
Gaussian error, $d = 0.2$ and $\theta_1 = 0.7$										
$\hat{d}$(mean)	0.0688	−0.1807	0.0218	−0.1968	−0.3365	−0.5121	0.0336	−0.2333	0.1074	0.0125
MSE	0.0561	0.1555	0.0549	0.1645	0.3074	0.5200	0.0889	0.2014	0.0449	0.0656
$\hat{\theta}_1$	0.5507	0.2668	0.5111	0.2471	0.0353	−0.2481	0.4965	0.1901	0.6015	0.5196
MSE	0.0662	0.2074	0.0636	0.2188	0.4911	0.9466	0.1198	0.2903	0.0418	0.0632
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, and $\theta_1 = 0.7$										
$\hat{d}$(mean)	0.0736	−0.1791	0.0309	−0.1953	−0.3381	−0.5139	0.0308	−0.2344	0.1020	0.0253
MSE	0.0515	0.1536	0.0506	0.1624	0.3076	0.5252	0.0894	0.2017	0.0482	0.0607
$\hat{\theta}_1$	0.5555	0.2635	0.5205	0.2440	0.0232	−0.2602	0.4914	0.1814	0.5982	0.5332
MSE	0.0631	0.2124	0.0588	0.2228	0.5115	0.9728	0.1262	0.3022	0.0491	0.0597

the estimates. The parametric methods presented the smallest MSE. There is no deterioration in the performance of the estimators under ARCH errors.

When $\theta > 0$, the picture of the estimation results changes dramatically. The bias of d is very large providing evidence that the biases of the parameters are directly related to the sign of the short-memory model.

When dealing with both short-memory parameters the picture is presented in Tables 5 and 6. The performance of the methods are very similar to the models previously considered. The biases are directly related to the sign and magnitude of the ARMA coefficients and the size of the bandwidth. The ML estimator loses a lot of its superiority for some AR and MA parameter combinations. The estimates of the ARFIMA parameters under Gaussian errors are similar to those seen elsewhere.

In general, the methods seem to be robust in the presence of ARCH errors and the bias and MSE become large in the presence of AR and/or MA parameters with values close to the boundary stationary conditions.

The next two tables show the estimation results from the ARFIMA(0, d, 0) and ARFIMA(1, d, 0) models with GARCH(1, 1) errors. For comparison purpose, our model is the same considered by Ling and Li (1997). From Tables 7 and 8, we notice similar performance with the previous cases. In the semiparametric class with the smallest bandwidth, the bias and MSE are generally small. By increasing the bandwidth, the biases of the empirical estimates increase substantially. The parametric $\hat{d}_{Fox}$ estimate has the smallest bias and MSE, however, its parametric method competitor performs poorly. The $\hat{d}_{Fox}$ estimates are very close to the

Table 5
ARFIMA(1, d, 1) model with Gaussian and GARCH(1, 0), errors, $d = 0.2$, $\phi_1 = -0.7, 0.7$, $\theta_1 = 0.2$, and $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
Gaussian error, $d = 0.2$, $\phi_1 = -0.7$, and $\theta_1 = 0.2$										
$\hat{d}$(mean)	0.1973	0.1151	0.1438	0.0942	0.0704	−0.1728	0.1918	0.0990	0.2167	0.1490
MSE	0.0372	0.0158	0.0275	0.0175	0.0251	0.1474	0.0694	0.0217	0.0206	0.0108
$\hat{\phi}_1$	−0.6981	−0.7152	−0.7113	−0.7180	−0.7242	−0.7537	−0.6860	−0.7202	−0.6984	−0.7033
MSE	0.0145	0.0034	0.0037	0.0036	0.0038	0.0526	0.0399	0.0034	0.0031	0.0028
$\hat{\theta}_1$	0.1927	0.0899	0.1259	0.0647	0.0305	−0.2969	0.1883	0.0653	0.2137	0.1484
MSE	0.0576	0.0275	0.0474	0.0302	0.0439	0.2852	0.1001	0.0393	0.0297	0.0183
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, $\phi_1 = -0.7$, and $\theta_1 = 0.2$										
$\hat{d}$(mean)	0.1794	0.1092	0.1295	0.0906	0.0677	−0.1831	0.1834	0.0974	0.1858	0.1422
MSE	0.0397	0.0191	0.0288	0.0190	0.0265	0.1561	0.0727	0.0251	0.0151	0.0123
$\hat{\phi}_1$	−0.7000	−0.7162	−0.7117	−0.7192	−0.7237	−0.7614	−0.6844	−0.7194	−0.7070	−0.7033
MSE	0.0132	0.0040	0.0048	0.0038	0.0045	0.0420	0.0485	0.0046	0.0070	0.0032
$\hat{\theta}_1$	0.1734	0.0845	0.1117	0.0623	0.0305	−0.3165	0.1828	0.0663	0.1805	0.1437
MSE	0.0611	0.0334	0.0514	0.0330	0.0471	0.3014	0.1040	0.0436	0.0265	0.0211
Gaussian error, $d = 0.2$, $\phi_1 = 0.7$, and $\theta_1 = 0.2$										
$\hat{d}$(mean)	0.3232	0.5304	0.2722	0.5145	0.3854	0.4201	0.3674	0.5782	0.1649	−0.0519
MSE	0.0603	0.1196	0.0313	0.1058	0.0355	0.0489	0.1048	0.1571	0.0582	0.1138
$\hat{\phi}_1$	0.5120	0.2685	0.5826	0.2966	0.5063	0.4634	0.4675	0.1875	0.6472	0.7860
MSE	0.1031	0.2386	0.0511	0.2059	0.0554	0.0782	0.1472	0.3204	0.0681	0.0337
$\hat{\theta}_1$	0.1285	0.0847	0.1504	0.0983	0.1829	0.1735	0.1244	0.0487	0.1094	0.0437
MSE	0.0313	0.0511	0.0222	0.0470	0.0249	0.0290	0.0419	0.0622	0.0380	0.0398
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, $\phi_1 = 0.7$, and $\theta_1 = 0.2$										
$\hat{d}$(mean)	0.3245	0.5371	0.2662	0.5172	0.3869	0.4199	0.3593	0.5828	0.1052	−0.0590
MSE	0.0581	0.1239	0.0297	0.1076	0.0361	0.0488	0.0921	0.1600	0.0674	0.1135
$\hat{\phi}_1$	0.5179	0.2549	0.5944	0.2961	0.5098	0.4736	0.4862	0.1792	0.6904	0.7984
MSE	0.1007	0.2589	0.0492	0.2131	0.0571	0.0763	0.1325	0.3317	0.0597	0.0314
$\hat{\theta}_1$	0.1386	0.0802	0.1586	0.1023	0.1899	0.1855	0.1387	0.0470	0.0984	0.0510
MSE	0.0326	0.0587	0.0244	0.0523	0.0296	0.0336	0.0389	0.0653	0.0429	0.0406

estimated parameter values given by Ling and Li (1997). In general, the estimates by semiparametric and the $\hat{d}_{Fox}$ approaches are not affected by GARCH error type. This result is consistent with those that have been presented in Ling and Li (1997) and Henry (2001a).

5. Concluding Remarks

Through the simulation studies this article shows that a class of parametric and semiparametric memory parameter estimators are robust with the presence of ARCH and GARCH errors. These empirical results evidence that the asymptotic normality results for the investigated estimation methods still hold for heteroscedastic errors. The biases are a consequence of the characteristics of the estimators and they depend on the structure of the ARFIMA model. In this context,

Table 6
ARFIMA(1, d, 1) model with Gaussian and GARCH, errors, $d = 0.2$, $\phi_1 = 0.2$, $\theta_1 = -0.7, 0.7$, and $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
Gaussian error, $d = 0.2$, $\phi_1 = 0.2$, and $\theta_1 = -0.7$										
$\hat{d}$(mean)	0.2141	0.2935	0.1578	0.2707	0.2487	0.3419	0.2039	0.3051	0.1035	0.0631
MSE	0.0382	0.0184	0.0261	0.0112	0.0059	0.0212	0.0677	0.0239	0.0689	0.0665
$\hat{\phi}_1$	0.1937	0.0911	0.2539	0.1155	0.1414	0.0407	0.2176	0.0803	0.2850	0.3274
MSE	0.0555	0.0281	0.0429	0.0193	0.0121	0.0309	0.0918	0.0340	0.0758	0.0703
$\hat{\theta}_1$	−0.7018	−0.7143	−0.6927	−0.7094	−0.7050	−0.7200	−0.7005	−0.7162	−0.6912	−0.7022
MSE	0.0041	0.0037	0.0040	0.0038	0.0036	0.0038	0.0046	0.0039	0.0038	0.0032
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, $\phi_1 = 0.2$, and $\theta_1 = -0.7$										
$\hat{d}$(mean)	0.2253	0.3014	0.1744	0.2807	0.2515	0.3453	0.2183	0.3133	0.0399	0.0796
MSE	0.0378	0.0201	0.0232	0.0129	0.0064	0.0223	0.0647	0.0257	0.1191	0.0578
$\hat{\phi}_1$	0.1893	0.0932	0.2443	0.1148	0.1478	0.0479	0.2093	0.0823	0.3549	0.3193
MSE	0.0553	0.0287	0.0399	0.0199	0.0126	0.0288	0.0886	0.0342	0.1191	0.0655
$\hat{\theta}_1$	−0.7012	−0.7116	−0.6915	−0.7071	−0.7031	−0.7160	−0.7001	−0.7133	−0.6902	−0.6998
MSE	0.0035	0.0031	0.0036	0.0033	0.0030	0.0031	0.0040	0.0033	0.0034	0.0031
Gaussian error, $d = 0.2$, $\phi_1 = 0.2$, and $\theta_1 = 0.7$										
$\hat{d}$(mean)	0.0883	−0.1313	0.0367	−0.1509	−0.2539	−0.3310	0.0586	−0.1767	0.1741	−0.0519
MSE	0.0500	0.1197	0.0485	0.1297	0.2227	0.2931	0.0835	0.1544	0.0646	0.1138
$\hat{\phi}_1$	0.1669	0.0667	0.1572	0.0314	0.2171	0.4149	0.1738	0.0923	0.0836	0.7860
MSE	0.0334	0.0677	0.0288	0.0716	0.1601	0.2479	0.0553	0.0922	0.0532	0.0337
$\hat{\theta}_1$	0.5482	0.2112	0.4879	0.1532	0.2196	0.3298	0.5204	0.1797	0.5580	0.0437
MSE	0.0879	0.2937	0.1009	0.3435	0.3426	0.2975	0.1146	0.3394	0.1486	0.0398
ARCH error with $\alpha_0 = 1$ and $\alpha_1 = 0.2$, $d = 0.2$, $\phi_1 = 0.2$, and $\theta_1 = 0.7$										
$\hat{d}$(mean)	0.0813	−0.1283	0.0330	−0.1492	−0.2516	−0.3328	0.0492	−0.1730	0.1313	0.0796
MSE	0.0519	0.1183	0.0497	0.1287	0.2209	0.2969	0.0864	0.1524	0.0621	0.0578
$\hat{\phi}_1$	0.1781	0.0690	0.1604	0.0362	0.2317	0.4367	0.1893	0.1038	0.0988	0.3193
MSE	0.0356	0.0625	0.0318	0.0688	0.1655	0.2634	0.0592	0.0934	0.0666	0.0655
$\hat{\theta}_1$	0.5530	0.2147	0.4878	0.1573	0.2310	0.3499	0.5266	0.1929	0.5342	0.6998
MSE	0.0814	0.2875	0.1026	0.3384	0.3355	0.2888	0.1076	0.3292	0.1719	0.0031

Table 7
ARFIMA(0, d, 0)-GARCH(1, 1) model, with $d = 0.3$ and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
GARCH error, with $\alpha_0 = 0.4$, $\alpha_1 = 0.3$, and $\beta_1 = 0.3$										
$\hat{d}$(mean)	0.3050	0.3099	0.2565	0.2912	0.2546	0.2629	0.3067	0.3111	0.3089	0.2909
bias	0.0050	0.0099	−0.0435	−0.0088	−0.0454	−0.0371	0.0067	0.0111	0.0089	−0.1591
sd	0.1916	0.0985	0.1468	0.0816	0.0642	0.0511	0.2341	0.1055	0.0566	0.0537
MSE	0.0366	0.0098	0.0234	0.0067	0.0062	0.0040	0.0547	0.0112	0.0033	0.0282

Table 8
ARFIMA(1, d, 0)-GARCH(1, 1) model, with $d = 0.3$, $\phi_1 = 0.5$, and sample size $n = 300$

Estimator	$\hat{d}_{Per}$		$\hat{d}_{Per\,S}$		$\hat{d}_{Lob}$		$\hat{d}_{Rob}$		$\hat{d}_{Fox}$	$\hat{d}_{ML}$
$g(n)$	$n^{0.5}$	$n^{0.7}$	$n^{0.5}$	$n^{0.7}$	$n^{0.7}$	$n^{0.8}$	$n^{0.5}$	$n^{0.7}$	–	–
	GARCH error, with $\alpha_0 = 0.4$, $\alpha_1 = 0.3$, and $\beta_1 = 0.3$									
$\hat{d}$(mean)	0.3227	0.4581	0.2734	0.4422	0.3538	0.4161	0.3247	0.4798	0.2862	0.1455
bias	0.0227	0.1581	−0.0266	0.1422	0.0538	0.1161	0.0247	0.1798	−0.0138	−0.1545
sd	0.1898	0.0949	0.1472	0.0786	0.0399	0.0205	0.2290	0.1007	0.1471	0.1652
MSE	0.0364	0.0340	0.0223	0.0264	0.0045	0.0139	0.0529	0.0424	0.0217	0.0512
$\hat{\phi}_1$(mean)	0.4697	0.3362	0.5179	0.3503	0.4391	0.3780	0.4702	0.3156	0.5002	0.6281
bias	−0.0303	−0.1638	0.0179	−0.1497	−0.0609	−0.1220	−0.0298	−0.1844	0.0002	0.1281
sd	0.1900	0.1070	0.1556	0.0916	0.0658	0.0555	0.2281	0.1111	0.1584	0.1520
MSE	0.0369	0.0382	0.0244	0.0308	0.0080	0.0180	0.0528	0.0463	0.0250	0.0395

the positive parameters of the AR and MA components contribute to a significant increase of the bias when their values are close to non stationary conditions. This can be explained by the shape of the spectral density when the AR and/or MA parameters are present in the model. Some findings presented here are in accordance with other that have recently appeared in the literature of volatility long-memory models.

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